

1970

# On determining the reliability of protective relay systems

Jack Duane Grimes  
*Iowa State University*

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ON DETERMINING THE RELIABILITY OF  
PROTECTIVE RELAY SYSTEMS

by

Jack Duane Grimes

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In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University  
Ames, Iowa

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## TABLE OF CONTENTS

	Page
INTRODUCTION	1
REVIEW OF THE LITERATURE	6
RELIABILITY OF ANALOG RELAYS	8
$q_{fail}$ — THE PROBABILITY OF A FAILURE TO OPERATE	13
$q_{nons}$ — THE PROBABILITY OF NONSELECTIVE ACTION	16
COST OF RELAY PROTECTION	18
APPLYING CLASSICAL RELIABILITY THEORY	19
PREDICTION OF PROTECTIVE RELIABILITY	25
Variation of $\lambda$ with Respect to Time	29
A NUMERICAL EXAMPLE	32
Determination of $q_{fail}$ from Operating Data	35
Determination of $q_{nons}$ from Operating Data	36
Determination of $Q_{fail}$ from Operating Data	41
Determination of $Q_{nons}$ from Operating Data	43
COMPUTER OPERATED PROTECTION RELIABILITY	47
CONCLUSIONS	51
BIBLIOGRAPHY	53
ACKNOWLEDGEMENTS	55

## LIST OF FIGURES

	Page
Figure 1. One line diagram of a section of a power system . . . . .	2
Figure 2. Failure probabilities for different schemes . . . . .	9
Figure 3. Reliability for an exponential density function . . . . .	20
Figure 4. Failure rate of an automatic reclosing device as a function of the number of operations . . . . .	21
Figure 5. Expected variation of the failure rates by months due to lightning . . . . .	31
Figure 6. Comparison of 1968 weather related disturbances (curve 1) with the 34 year average incidence of lightning (curve 2) . . . . .	34
Figure 7. Variation of $\lambda_{\text{nonS}}$ by months for CECO - 1968 . . . . .	43

## LIST OF TABLES

	Page
Table 1. Behavior of Q for various values of $\lambda$ for T = 1 week . .	22
Table 2. A description of the statistical quantities used in Equations 2, 5, 11 and 12 . . . . .	28
Table 3. Number of days of damaging lightning in Illinois . . . .	29
Table 4. Summary of relay success and failure for 1968 by months . . . . .	34
Table 5. System data for determining $q_{nons}^Y$ . . . . .	38
Table 6. Data for the determination of $q_{nons}$ . . . . .	39
Table 7. Statistics describing $q_{nons}$ for line 2102 . . . . .	40
Table 8. Statistics describing $q_{nons}$ for line 1321 . . . . .	40
Table 9. Unreliability of nonselective action, $Q_{nons}^{sys, month}$ by months. Note that $Q_{nons}$ for the best months (0.865) is close to unity . . . . .	44
Table 10. $Q_{nons}^{2101}$ for each month for line 2101 for 1968 . . . . .	45

## INTRODUCTION

Electric power companies have a substantial capital investment in generating stations, transmission lines and distribution networks. To protect this expensive equipment, protective relays are used to detect the presence of faults (short circuits and other abnormal conditions) which would damage the equipment or otherwise interfere with the normal operation of the rest of the power system. These relays have a pair of electrical contacts which close and energize an auxiliary relay to handle the heavy currents necessary to operate circuit breakers. The circuit breakers disconnect the faulted equipment thereby isolating it from the rest of the power system.

As power systems grow in complexity, relays play a more and more important role in the removal of faulted equipment. Since system stability becomes more critical, relays are required to operate faster and without errors. These requirements have resulted in relay manufacture being characterized as an "art".

Relays operate by sensing voltages, currents, phase angle and other quantities which will enable them to distinguish a fault from normal conditions on a particular piece of equipment. The normal state of the relay is passive since faults are relatively rare. The relay is required not to operate unless the fault which does occur, is on the particular piece of equipment being protected. Figure 1 below is a one line diagram of a section of a power system. The square boxes are circuit breakers, the heavy lines are connection busses and the thin lines are transmission lines. The dashed lines surround a section of transmission line to be

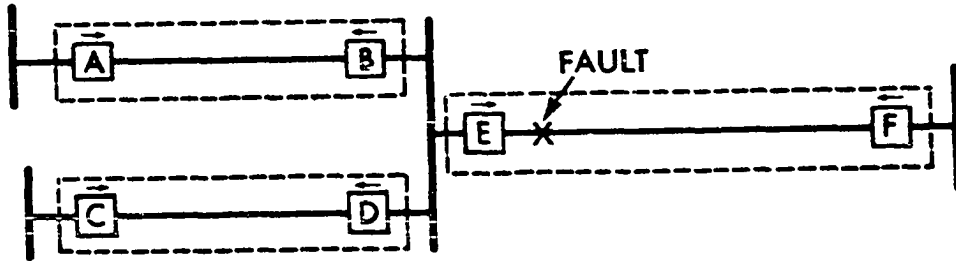


Figure 1. One line diagram of a section of a power system

protected and indicate the "primary protection zone" for the relays on either end of the line. Should a fault occur on line EF, two things must happen.

- 1) The relays at each end of the faulted line (indicated by arrows) sense this fault and open circuit breakers E and F.
- 2) The other relays such as are at A and B are expected to not operate.

For this same fault on line EF, back-up relays are necessary to remove the fault should a failure occur in the relays at E or F or in the circuit breaker at E or F. These back-up relays are usually of the same type as the primary relays and are enabled after a time delay long enough for the primary system to normally isolate the fault.

The characteristic of relays which indicates their ability to detect faults they were supposed to detect and to ignore other faults is called selectivity. Selectivity may be gained by the use of particular relays, by a particular configuration of relays and communication



channels or a combination of the two.

One of the first ways of providing selectivity as well as back-up was through the use of a directional overcurrent relay with an inverse time characteristic. This relay senses currents flowing in a certain direction (down the line) above a preset magnitude. When the line current exceeds this preset threshold, the relay responds with a time delay inversely related to the magnitude of the current. For relays closer to the fault, larger fault currents are seen and these relays operate first providing selectivity. The more distant relays will operate if the closer relays fail to isolate the fault, providing natural back-up.

As relays were required to operate faster, instantaneous relays were used for primary relaying, and with variable time delay for back-up relaying. Two common types are instantaneous overcurrent and impedance relays. Impedance relays make a continuous (analog) computation of the complex ratio of voltage to current. The impedance seen by the relay is the impedance of the transmission line plus the impedance of the load connected to the far end. This quantity will be greater than the impedance of the line alone unless a fault occurs on the line. These relays are inherently directional.

For a fault close to the end of the line EF, as indicated in Figure 1 a problem arises. How does the set of relays at F sense this fault and yet not respond to a fault located on line AB near circuit breaker B? Electrically, these two locations are very close to each other. One solution is to set the relay at F to sense a fault along as well as beyond the end of the line and provide a signal from the

other end of the line to block the operation of breaker F if the fault is not on line EF. This blocking signal is present unless the relays at E also sense a fault. This scheme is referred to as an overreaching-blocking relay scheme.

The alternative to an overreaching-blocking scheme is an under-reaching-permissive scheme. In this scheme if one of the relays senses the fault, the related breaker is tripped and this trip is transferred to all other circuit breakers connecting this line to the rest of the power system.

The two schemes above detect the presence of faults based on "local" information. Some relaying schemes in use utilize information from both ends of the line. This differential protection requires a communication channel for information as well as for transfer trip or blocking signals.

One of the major problems of reliable relay operation is the security of the communications channels. These channels may be leased wire, private wire, power line carrier current, public microwave or private microwave. The security of the channels is part of a broader security requirement to prevent spurious operations of any type.

The requirement of positive detection of all faults by the proper relays and the prevention of unwanted trips is generally classed as a reliability problem, or, stated another way, the probability of all operations being correct. This "probability of correct operations" is a definition in a wide sense. If a faulted line is cleared, either by primary relaying or one of several back-up relays, and if there are no spurious operations, the operations could be called correct. In this thesis a more narrow definition will be adopted, one of fault-free

performance of the primary protection system whether it be a group of relays protecting a certain transmission line or a digital system protecting one or more transmission lines. Since we will compare the reliability of existing equipment with that of a proposed digital system the primary protection system will be defined to include:

- 1) primary zone relays
- 2) station batteries
- 3) mounting racks
- 4) interconnections between relays.

Specifically excluded are current and potential transformers, circuit breakers and the circuit breaker trip mechanisms.

The purpose of this thesis is to derive two methods of calculating the reliability of existing protective relays. These two techniques are demonstrated by way of a numerical example. The numerical results are discussed and compared with the reliability of a computer operated protection system.

## REVIEW OF THE LITERATURE

An extensive literature search was made in the area of power system reliability. Many papers were found on the general topic of reliability and many others on the problems of determining the reliability of power systems. Most of the pertinent papers dealt with the prediction of system reliability with respect to generator and line outages. Only three papers, all published by Soviet authors (4, 11, 13) were related to the topic of protective relay reliability. These papers were found translated into English in Electric Technology U.S.S.R.

The paper by Smirnov (11) presents the time independence of the relay's inherent reliability and gives basic ground rules for determining this reliability in a laboratory environment. He then discusses the effect of field conditions on this reliability.

The paper by Fabrikant (4) presents the concept of the double nature of relay reliability. The equations as stated in (4) are not without errors. However, the approach adopted in the first portion of this thesis is based on the concepts presented there.

The paper by Zul' and Kuliev (13) is useful in that they have presented test data on a Soviet automatic protective device and have shown that it follows the classical failure modes described by an exponential time to failure.

A previous literature review brought to light several papers (2, 5, 7, 9, 12) on computer operated substation protection systems. All of the computer systems have basically the same hardware requirements and the reliability of these systems is discussed in

this thesis.

A related paper by Corduan and Eddy (3) discusses three ways of providing stored energy to isolate the hardware from power supply perturbations. A reliable source of power is necessary for a computer operated protection system.

## RELIABILITY OF ANALOG RELAYS

Determination of the reliability of a protective relay or relaying system is unique. The ordinary concept of probability of failure-free operation does not directly apply. Instead, two reliability quantities must be considered,  $q(x)$ <sup>1</sup>, the probability of a device failing to operate properly in the presence of a fault it was set to detect and  $q(y)$ , the probability of a device operating in the presence of an external fault which it is supposed to ignore (4). In addition,  $q_{fail}$  will refer to the probability of a group of relays or relay systems failing to operate and  $q_{nons}$  will denote a group of relays or relay systems operating non-selectively.

Assume that a fault outside the primary zone of the relay<sup>2</sup> produces an unwanted trip signal, i.e., signals are produced when unnecessary. To lower the probability of this nonselective action, we can place another relay in parallel and connect them by a perfect logical AND element (see Figure 2, case A). The probability of nonselective action is

$$q_{nons} = [q(y)]^2$$

so that

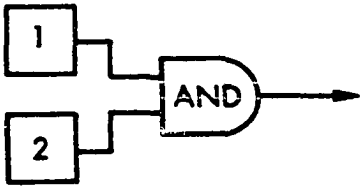
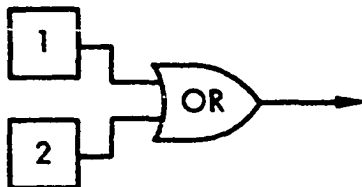
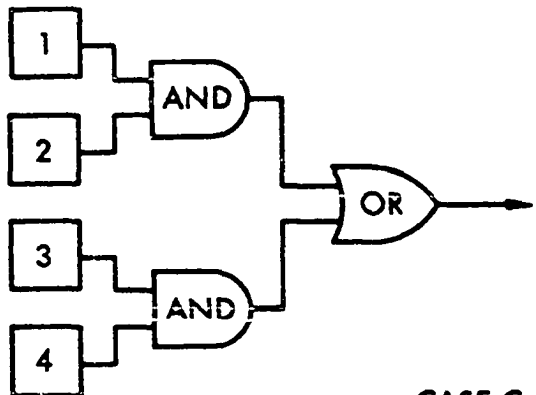
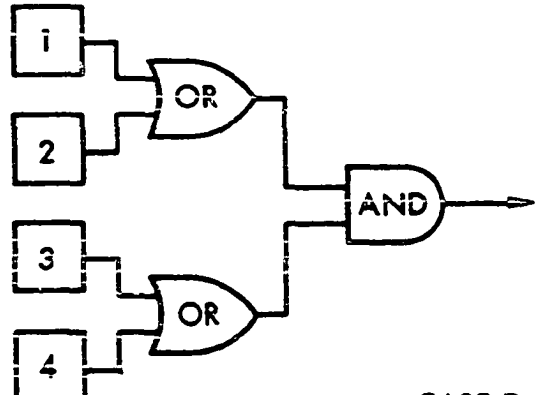
$$q_{nons} \text{ is less than } q(y)$$

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<sup>1</sup>For the sake of brevity, this paper will use the notion of unreliability,  $q$ , which is related to the reliability,  $r$ , by the equation  $q = 1 - r$ .

<sup>2</sup>The technique presented here applies equally well to a particular device (a relay) or to a group of relays acting as a unit. Thus, relay and relay systems can usually be used synonymously.

Figure 2. Failure probabilities for different schemes

SCHEME	PROBABILITY	
	NONSELECTIVE OPERATION $q(y)$	FAILURE $q(x)$
 <p style="text-align: center;">CASE A</p>	$q(y)^2$	$2q(x) - q(x)^2$
 <p style="text-align: center;">CASE B</p>	$2q(y) - q(y)^2$	$q(x)^2$
 <p style="text-align: center;">CASE C</p>	$2q(y)^2 - q(y)^4$	$[2q(x) - q(x)^2]^2$
 <p style="text-align: center;">CASE D</p>	$[2q(y) - q(y)^2]^2$	$2q(x)^2 - q(x)^4$



since  $q(y)$  is less than one.

However, this relay can also fail to operate in the presence of a fault, i.e., no trip signal is produced when one is necessary. Connecting two relays in parallel through the logical AND element results in a probability of failure,

$$\begin{aligned} q_{\text{fail}} &= 1 - [1 - q(x)]^2 \\ &= 2q(x) - [q(x)]^2 \end{aligned}$$

which is greater than  $q(x)$  since  $q(x)$  is less than one. Thus  $q_{\text{nons}}$  is lowered at the expense of raising  $q_{\text{fail}}$ .

For example, let  $q(x) = 0.05$  and  $q(y) = 0.1$ . If we connect two relays with these characteristics together with a perfect AND element,

$$\begin{aligned} q_{\text{nons}} &= [q(y)]^2 \\ &= 0.1^2 \\ &= 0.01 \end{aligned}$$

which is a factor of ten improvement over each relay alone. However,

$$\begin{aligned} q_{\text{fail}} &= 2q(x) - q(x)^2 \\ &= 2(0.05) - (0.05)^2 \\ &= 0.10 - 0.0025 \\ &= 0.0975 \end{aligned}$$

which is worse than each relay alone by a factor of about two. This example points up two things: 1) Both failure modes must be evaluated to compare different schemes; 2) If  $q(x)$  and  $q(y)$  are very different (as they may easily be), a trade-off is possible between  $q_{\text{nons}}$  and  $q_{\text{fail}}$  to make them more nearly equal.

The scheme in case B of Figure 2, which is commonly used in power systems, accomplishes opposite of case A, i.e., case B improves  $q_{fail}$  at the expense of degrading  $q_{nons}$ . The schemes of case C and case D improve (lower) both  $q_{nons}$  and  $q_{fail}$  by using 4 relays and 3 logic elements, but only if  $q(y)$  and  $q(x)$  are less than 0.382. If  $q(y)$  and  $q(x)$  are greater than 0.382 it is not possible (4) to simultaneously improve  $q_{nons}$  and  $q_{fail}$ , a fact resulting from the double nature of the reliability of protective relays. The practice of making a reliable system from unreliable parts is applicable here only insofar as their inherent reliability is sufficiently good.

If a relay fails so that the failure could lead to a failure of the protection, this does not imply that a failure of the protection actually takes place. For the protection to fail, the line also must be faulted.

Different from this is the nonselective failure where a trip signal is sent for a fault external to the primary zone of the relay. Soviet experience indicates (4) that almost all of the nonselective actions of the relay occur simultaneously with an external fault. Very rarely does the relay send a trip signal all by itself.

$q_{fail}$  — THE PROBABILITY OF A FAILURE TO OPERATE

With existing relaying systems, the relays do not operate unless a fault occurs. Since faults are relatively rare, the relay spends nearly all of its life in a passive state and doesn't see fault magnitude quantities. This is all right, of course, because the relay needs to operate only when a fault occurs. However, the problem is that one doesn't know if the relay will operate correctly until the next fault occurs, at which time the relay is called upon to operate. Thus the probability of failure depends upon the frequency of faults occurring within the relays primary protective zone. This is an inherent disadvantage. While the relay is in its passive state, there is no way to predict whether or not the relay will operate should a fault occur. A computer operated protection scheme, on the other hand, has a probability of failure which is less dependent upon the frequency of occurrence of faults on the line and allows convenient prediction (via self-testing) of successful operation should a fault occur.

For existing relay systems, the probability of a failure to operate  $q_{fail}$  is dependent upon two quantities,  $q(x_a)$  and  $q(x_b)$ . If  $x_a$  represents the event of a fault in the primary zone, then  $q(x_a)$  represents its probability of occurrence. Similarly,  $x_b$  represents the event of a failure of the relay and its probability of occurrence is  $q(x_b)$ . The probability of failure to operate of the relays on line  $\gamma$  is given by the intersection of the two events

$$q_{fail}^{\gamma} = q(x_a \text{ and } x_b) \quad (1)$$

When the probability of one event is dependent on the occurrence of

another, a conditional probability is involved. The applicable relation (10) for the intersection of two events in Equation 1 is

$$q_{fail}^Y = q(x_a)q(x_b|x_a) \quad (2)$$

The probability  $q(x_b|x_a)$  does not equal  $q(x_b)$  unless the events are independent. For existing relaying systems, events  $x_b$  and  $x_a$  are not independent, and Equation 2 cannot be simplified. For existing relays, we may define the probability of a failure in the relay as a function of the number of operations the relay performs. Of course, the number of operations in a year depends upon  $q(x_a)$ , the probability of faults on the protected line in a year. On the average, the relay will fail on a certain percentage of the total number of operations.

It is common practice in reliability work to use time (or a time period T) as a basic index instead of the number of operations. Due to this use of time as the basic index, a relay placed in service on a line with greater fault-proneness (faultability) will exhibit a shorter average (mean) time between failures (MTBF) than a similar relay placed in service on a less fault-prone line. This change in MTBF isn't due to any inherent change in the reliability of the relay, but instead is due to the choice of MTBF as the index of performance.

Note that MTBF is an excellent index to use for the reliability of a digital computer as the failures are indeed time independent. We should also note that a common index is necessary to compare the reliability of the two systems. Hence, we shall use the unreliability based on MTBF to measure both systems in order to make a comparison of the relative reliability of each.

The ratio of the number of faults internal to the number of faults external to the primary zone of the relay can be related to a "level of exploitation" (11) of the relay. The inherent reliability of the relay itself is independent of its placement, but the probability of a given relay operating successfully for a time T depends on the faultability of the line as well. Suppose we have a relay that fails to operate three per cent of the time,  $q(x_b | x_a) = 0.03$ . If placed on a line with 100 faults/yr, the mean-time-between-failures (MTBF) is  $365/3$  or 122 days. If this same relay is placed on a line with 200 faults per year, the MTBF is 61 days. For a given period of time, if a relay sees more faults, it will fail more often since  $q(x_b | x_a)$  is a statistical parameter which is a constant for each relay. In summary, we note two things:

- 1) the probability of successful operation (reliability) is dependent on the faultability of line, since  $r = 1 - q_{fail}$ .
- 2) The use of time as a base, i.e., MTBF, is not completely satisfactory for existing protection relay systems.

$q_{\text{nons}}$  — THE PROBABILITY OF NONSELECTIVE ACTION

There are two ways a relay can act nonselectively. First, the relay may act in a nonfault condition. This may be due to a slow failure of the device such as from age or environment or due to a disturbance of the relay. Second, the relay may act due to a fault occurring outside its primary zone (an external fault) simultaneously with a relay failure. Soviet experience (4) shows that most nonselective actions of relay protection systems occur in conjunction with external faults. Based on operating data presented later this is not true for U.S. power systems, where most nonselective acts are caused by security failures.

Referring to Figure 1, the relays at A or B should not operate for a fault on line EF. If they do operate, it is a nonselective operation. The probability of this type of failure is

$$q_{\text{nons}}^{\text{AB}} = q(y_s^{\text{AB}}) + q(y_a^{\text{EF}})q(y_b^{\text{AB}}|y_a^{\text{EF}})$$

where  $q(y_s^{\text{AB}})$  is the probability of a nonselective operation due to a security failure on line AB,  $q(y_a^{\text{EF}})$  is the probability of a fault on line EF and  $q(y_b^{\text{AB}}|y_a^{\text{EF}})$  is the probability of the relays on line AB responding to the fault on line EF when it occurs. In general, let  $q(y_s^{\text{Y}})$  equal the probability of nonselective action occurring on line  $y$  when there are no external faults.<sup>1</sup> If  $y_a$  represents the event of a

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<sup>1</sup>This should include nonselective operations due, for example, to a workman drilling on the relay panel as well as other nonfault related events.

fault, external to line  $\gamma$  then the probability of external faults on lines 2, 3,  $\dots$ ,  $m$  equals  $q(y_a^2)$ ,  $q(y_a^3)$ ,  $\dots$ ,  $q(y_a^m)$ . Let the probability of a relay on line  $\gamma$  failing so as to act nonselectively in the presence of external faults (a conditional probability) equal  $q(y_b^\gamma|y_a^2)$ ,  $q(y_b^\gamma|y_a^3)$ ,  $\dots$ ,  $q(y_b^\gamma|y_a^m)$ . The probability of nonselective operations on line  $\gamma$  is the product of the probability of no security failures and the probabilities of no fault related failures. Stated mathematically for line  $\gamma$ ,

$$q_{\text{nons}}^\gamma = 1 - \left[ 1 - q(y_s^\gamma) \right] \left[ 1 - q(y_a^2)q(y_b^\gamma|y_a^2) \right] \left[ \dots \right] \left[ 1 - q(y_a^N)q(y_b^\gamma|y_a^N) \right]. \quad (3)$$

If  $q_{\text{nons}}^\gamma$  is 0.1 or less, we can use an approximation for a small loss in precision.

$$q_{\text{nons}}^\gamma \approx q(y_s^\gamma) + q(y_a^2)q(y_b^\gamma|y_a^2) + \dots + q(y_a^N)q(y_b^\gamma|y_a^N). \quad (4)$$

Then for line  $\gamma$ ,

$$q_{\text{nons}}^\gamma \approx q(y_s^\gamma) + \sum_{\substack{m=1 \\ m \neq \gamma}}^N q(y_a^m)q(y_b^\gamma|y_a^m). \quad (5)$$

## COST OF RELAY PROTECTION

From the above it is clear that the probability of nonselective action,  $q_{\text{nons}}$ , as well as the probability of failure to act,  $q_{\text{fail}}$ , are dependent on the faultability of the lines they are protecting as well as the relay's inherent reliability. Due to these dependencies, one cannot simply add  $q_{\text{nons}}$  and  $q_{\text{fail}}$  together to obtain a composite unreliability because the costs associated with nonselective action and failure to act usually differ. Stated mathematically, using  $C$  to indicate the relative cost per operation,

$$\text{Cost} = C_{\text{fail}} q_{\text{fail}} + C_{\text{nons}} q_{\text{nons}}. \quad (6)$$

Naturally the best type of back-up protection then is that which reduces both  $q_{\text{fail}}$  and  $q_{\text{nons}}$ . The cost is then reduced independently of the values of the individual quantities. It can also be seen that adding an identical parallel protection system may or may not improve the protection, since both  $q_{\text{nons}}$  and  $q_{\text{fail}}$  are affected.

Operating data for a given protection may indicate that  $q_{\text{nons}} = 0.1$  and  $q_{\text{fail}} = 0.01$ . If  $C_{\text{fail}}$  and  $C_{\text{nons}}$  are known or can at least be approximated, then Equation 6 above will give the cost per operation. If two of these systems are connected together with a perfect logical AND element,  $q_{\text{nons}}$  will decrease,  $q_{\text{fail}}$  will increase and the cost may go up or down depending on the relative magnitudes of  $C_{\text{nons}}$  and  $C_{\text{fail}}$ . Thus, it is possible to evaluate different schemes and to make a decision as to which one is better (from the standpoint of lower costs).



## APPLYING CLASSICAL RELIABILITY THEORY

Classical reliability theory is often concerned with random failures described by an exponential density function,  $f(t)$  similar to Figure 3 except for the scale factor  $1/\lambda$ .

$$f(t) = \frac{1}{\lambda} e^{-\lambda t}$$

The distribution of failures is given by  $F(t)$ .

$$F(t) = \int_0^t \frac{1}{\lambda} e^{-\lambda x} dx$$

$$F(t) = 1 - e^{-\lambda t}$$

where  $x$  is a dummy variable. Since the integral of  $f(t)$  from  $t = 0 \rightarrow \infty$  is unity, the probability of one or more failures from  $t = 0 \rightarrow T$  is given by

$$Q(T) = \int_0^T f(t) dt$$

$$Q(T) = 1 - e^{-\lambda T}$$

The probability of success  $R$  is related by  $R = 1 - Q$ , so

$$R(T) = e^{-\lambda T}.$$

Figure 3 shows how  $R(T)$  varies as a function of  $\lambda T$ . If the failure rate  $\lambda$  is constant then the equation for  $R(T)$  above will also give the probability of success for any time interval,  $T$ .

There are usually three distinct types of failures. Early in the lifetime of a device,<sup>1</sup> failures are often due to initial weakness or defects, weak parts, bad assembly, etc. During the middle period of device operation fewer failures take place, and the failure rate,  $\lambda$ , is

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<sup>1</sup>Here again, device may indicate a relay or a group of relays acting as a unit.

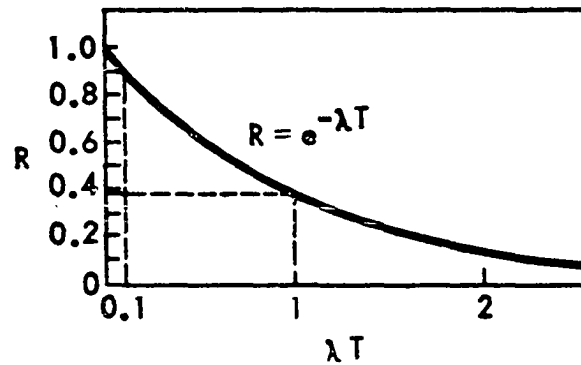


Figure 3. Reliability for an exponential density function

constant with respect to time. (For relays,  $\lambda$  is constant with respect to the number of operations.) Since the cause of these failures is difficult to determine, these middle-life failures are characterized as random events. As the device wears out, the failure rate,  $\lambda$ , increases again. In a well-designed and tested relay, the burn-in phase may be small or even nonexistent. We will make the usual assumption that our data reflects devices which are all in the normal, middle-life phase where failures are random and their failure rates ( $\lambda$ 's) are constant.

Figure 4 shows the results from Soviet tests (13) on an automatic reclosing device. The vertical axis indicates the failure rate per operation. The horizontal axis indicates the number of operations. The burn-in, middle, and wearout phases are easily recognized and are denoted by I, II and III respectively. This curve follows the general "bathtub" curve which is so well known in reliability theory (10).

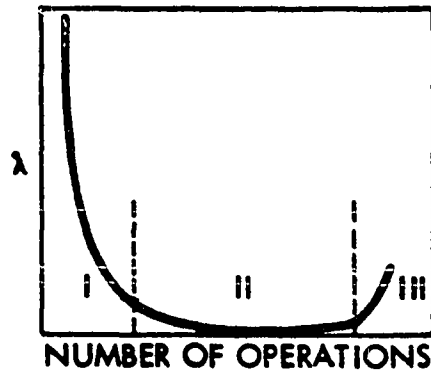


Figure 4. Failure rate of an automatic reclosing device as a function of the number of operations

The reliability of a device which meets the stated conditions is given by

$$R = e^{-\lambda T} \quad (7)$$

where  $\lambda$  is the failure rate or the number of failures per unit time during the middle life and  $T$  is the time period involved.  $R$  stands the probability of success (no failures) for the time period  $T$ . The unreliability or probability of failure is given by

$$Q = 1 - e^{-\lambda T} \quad (8)$$

where  $\lambda$  and  $T$  are as defined above and  $Q$  is the probability of one or more failures during the time period  $T$ .

An example is in order. Let us pick a time period such as  $T = 1$  week. By selecting various values for  $\lambda$ , we can see the behavior of  $Q$ , in Equation 8, summarized below in Table 1.

Table 1. Behavior of Q for various values of  $\lambda$  for T = 1 week

Q	$\lambda$ (Failures per week)	MTBF = $\frac{1}{\lambda}$
0	0	$\infty$
0.01	0.010	100 weeks
0.095	0.10	10 weeks
0.393	0.50	2 weeks
0.632	1.0	1 week
0.99965	10.0	0.1 week

As we can see from Table 1, the probability of a failure in a one week period, Q, is about 10% when there is an average of one failure per 10 weeks (MTBF = 10). This example demonstrates a very important property of Equation 8. In order to obtain a meaningful measure of the probability of failure it is imperative that Q be specified for a specific time period and the appropriate  $\lambda$  used for that time period. When using Equation 8 to determine the probability of failure during long time periods, a difficulty arises. It is assumed that the events giving rise to the failures described are randomly distributed with respect to time (during the time period T). If the time period is long, the failure rate may vary during the time period. However,  $\lambda$  can be assumed constant over some set of shorter time periods which make up the longer time period. In this case the  $\lambda$  in Equation 8 represents the arithmetic mean

of the  $\lambda$ 's which described the failure rates over this set of shorter time periods. To say it another way, Equation 8 assumes  $\lambda$  is constant over the time period  $T$ . When Equation 8 is applied to protective relaying reliability the failure rate  $\lambda$  is anything but constant when a period of one year is involved.

Rather than use Equation 8 as is, let us assume that  $\lambda$  remains constant only during each month of the year and develop the appropriate expressions to be used later. Let  $\lambda_i$  represent the failure rate for one month. Then the probability of no failures for January,  $R_J$ , is given by

$$R_J = e^{-\lambda_J T_M}$$

where  $T_M =$  one month. Similarly, the reliability  $R_i$  for any month  $i$  is given by

$$R_i = e^{-\lambda_i T_M} \quad (9)$$

The probability of no failures for January through December is

$$\begin{aligned} R_{\text{one year}} &= \prod_i R_i \quad i = 1, 2, \dots, 12 \\ &= \prod_i e^{-\lambda_i T_M} \quad i = 1, 2, \dots, 12 \\ &= e^{-(\lambda_1 + \lambda_2 + \lambda_3 \dots + \lambda_{12}) T_M} \end{aligned}$$

rearranging,

$$R_{\text{one year}} = e^{-\frac{(\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_{12})(12T_M)}{12}}$$

Also,

$$Q_{\text{one year}} = 1 - e^{-\frac{(\lambda_1 + \lambda_2 + \lambda_3 \dots + \lambda_{12})(12T_M)}{12}} \quad (10)$$

Comparing the right side of Equation 10 with Equation 8 above, we can see that

$$12T_M = T_Y$$

or a period of one year and that

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 \cdots + \lambda_{12}}{12}$$

which is the arithmetic mean of the monthly failure rates. The simple relation of Equation 10 enables one to derive yearly reliability quantities from statistics compiled monthly. It is important to note that there is likely to be a large variance associated with the monthly  $R_i$  given by Equation 9. Equation 10 represents an "averaging" of the monthly data. This is due to the nature of the application of classical reliability theory to power system protection.

Since there are two failure modes present, failure to operate and nonselective operation, there are two applicable expressions for R (or Q), one for each of the failure modes. That is, Equation 8 above gives rise to two equations:

$$Q_{\text{fail}} = 1 - e^{-\lambda_{\text{fail}} T} \quad (11)$$

$$Q_{\text{nons}} = 1 - e^{-\lambda_{\text{nons}} T} \quad (12)$$

In general,  $Q_{\text{nons}}$  and  $Q_{\text{fail}}$  will be different because they arise from two failure modes.

## PREDICTION OF PROTECTIVE RELIABILITY

The typical reliability problem concerns a system which is activated and performs its task until it fails. The system does not operate again until it is repaired. Furthermore, it is usually easy to tell when the system has failed since the output ceases (e.g. a communications systems).

A protective relaying system is "activated" when a fault or line disturbance occurs. The normal state of a relay can be assumed passive since the effects of age and environment are usually minimized by careful design.

Due to the extremely short period of operation it is not possible to even consider repair of the relay at the time of failure. For this and other reasons, some form of back-up (standby) protection is required. Since the back-up relays are also unreliable, we should provide back-up protection for the back-up relays, and so forth. Even if infinite funds were available, it will not be possible to reduce the probabilities of failure to zero. This is a direct result of the interconnection complexity encountered in implementing  $N$  back-up schemes. These  $N$  schemes must be connected together with logical elements (as in Figure 2) which are not failure free. If for no other reason, a graph of the costs (see Equation 6) associated with the probabilities of failure as a function of the number of back-up schemes,  $N$ , will exhibit a definite minimum.

A typical approach to the prediction of reliability has been to describe the probability of success in terms of the probabilities of success of the individual components. The procedure is to describe in closed form, by using logic equations, all of the possible combinations

for success (or failure) of the system and their probabilities. This technique has been proposed for protective relaying systems (11), however, the technique has several practical drawbacks. 1) Due to the complexity of modern relaying systems the logic equations become difficult to formulate. 2) Calculation of  $q_{fail}^{\gamma}$  and  $q_{nons}^{\gamma}$  by combinational methods is not possible since the individual  $q(x_b | x_a)$  and  $q(y_b^{\gamma} | y_a^m)$  for each relay are not generally known.<sup>1</sup>

The approach adopted here is to apply both the conventional description of Equations 2 and 5 and the classical description given by Equations 11 and 12. A subtle but important difference exists between the two techniques for evaluating relay performance. The description given by Equations 2 and 5, repeated below,

$$q_{fail}^{\gamma} = q(x_a)q(x_b | x_a) \quad (2)$$

$$q_{nons}^{\gamma} = q(y_s^{\gamma}) + \sum_{\substack{m=1 \\ m \neq \gamma}}^N q(y_a^m)q(y_b^{\gamma} | y_a^m) \quad (5)$$

is the probability of failure of the set of relays on line  $\gamma$  each time a fault or nonselective operation occurs on the power system. This description meshes quite well with several intuitive descriptions currently being used by electric utilities to characterize relay performance. Using Equations 2 and 5 and the appropriate data, the probabilities of failure given a fault has occurred,  $q(x_b | x_a)$  and

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<sup>1</sup>Values of  $q(x_b | x_a)$  and  $q(y_b^{\gamma} | y_a^m)$  seem to be unknown even by US relay manufacturers. The implication is that the appropriate statistics would have to be determined in the lab, which is not a pleasant prospect.



$q(y_b^Y | y_a^m)$ , can be calculated for each set of relays in addition to  $q_{fail}^Y$  and  $q_{nons}^Y$ . Since  $q(x_b | x_a)$  and  $q(y_b^Y | y_a^m)$  are independent of time and geographical placement of the relays and represent the probabilities of failure of each relay system acting as a unit, they are extremely useful quantities for comparison of different relaying schemes.

In contrast, the unreliabilities given in Equations 11 and 12, repeated below,

$$Q_{fail} = 1 - e^{-\lambda \text{ fail}^T} \quad (11)$$

$$Q_{nons} = 1 - e^{-\lambda \text{ nons}^T} \quad (12)$$

represent the probabilities of one or more failures in the time period T. The time period T can be interpreted as the time between routine maintenance checks. This interpretation of T doesn't suit relays very well because they spend most of their life in the passive state and it is difficult if not impossible to determine a priori if the relays will fail the next time a fault or spurious signal occurs.

If we think in terms of a substation computer operated protection system, however, the concept of routine maintenance and the ability to predict the future success of fault detection are very relevant. At this point it is anticipated that the  $Q_{fail}$  and  $Q_{nons}$  of a computer operated protection system will be so low that other previously neglected contributors to unreliability will now dominate the protection system unreliability.

Depending on how Equations 11 and 12 are used in conjunction with the protective system operating data, four pair of Q's can be calculated:

- 1)  $Q_{fail}$  and  $Q_{nons}$  for all the relays on the power system for

each month.

- 2)  $Q_{fail}$  and  $Q_{nons}$  for all the relays on the power system "averaged"<sup>1</sup> for the year.
- 3)  $Q_{fail}$  and  $Q_{nons}$  for each set of line relays acting as a unit for each month.
- 4)  $Q_{fail}$  and  $Q_{nons}$  for each set of line relays acting as a unit "averaged" for the year.

The statistical quantities in Equations 2, 5, 11, 12 are enumerated in Table 2 below.

Table 2. A description of the statistical quantities used in Equations 2, 5, 11 and 12.

Quantity	Description
$Y_{fail}$	Number of failures to act in the presence of a fault per time period.
$Y_{nons}$	Number of nonselective actions due to security failures or external faults per time period.
$q(y_s^Y)$	The probability of a security failure occurring on line $\gamma$ .
$q(x_a)$	The probability of a fault on the system occurring on line $\gamma$ .
$q(x_b   x_a)$	The probability that the line relays will fail to act given the fault, $x_a$ .
$q(y_a^m)$	The probabilities of a fault occurring on line $m$ .
$q(y_b^Y   y_a^m)$	The probabilities of the line relays on line $\gamma$ operating nonselectively given that a fault has occurred on line $m$ .

<sup>1</sup> See Equation 10.

To summarize, the predictive approach adopted here is to characterize mathematically the two failure mechanisms using two different descriptions. Then, one can interpret (untangle) the operating statistics and calculate useful reliability indexes with which to judge and compare the performance of line relays and of the protective system as a whole.

#### Variation of $\lambda$ with Respect to Time

Let us digress for a moment and investigate the expected variation of both  $\lambda$ 's with respect to time. Since a numerical example later in this thesis uses data from a major Illinois electric utility it is appropriate to discuss the incidence of lightning in Illinois.

By coincidence, an excellent paper exists (1) specifically concerned with the incidence of damaging lightning in Illinois for the period 1914-1947. All the remarks herein refer to conditions only in Illinois. Table 3 below shows the number of days of damaging lightning by months.

Table 3. Number of days of damaging lightning in Illinois

Month	Total # of days	Fraction of total
January	2	.0043
February	8	.0173
March	17	.0367
April	20	.043
May	45	.097
June	94	.203
July	114	.246
August	108	.233
September	36	.0777
October	17	.0367
November	2	.0043
December	0	0
TOTAL	463	

As expected, the peak of activity occurs in the summer months. Over 2/3 of the days experiencing damaging lightning occur during June, July and August. In the May 1 - September 15 period, 80% of all days with damaging lightning occurred.

The two months with the highest lightning frequency are July and August, while the months with the greatest number of thunderstorm days are May and June. No solid reason exists to explain this disagreement between two such highly correlated weather phenomena. One of several possible explanations (1) is simply that thunderstorms occurring in May and June do not produce as many or as strong cloud-to-ground discharges as they do in July and August. It is also interesting to note that the maximum number of occurrences of lightning during the day was found in the early afternoon between 1 p.m. and 4 p.m., local time.

The year to year variance in the data of Table 3 can be summarized by noting that the maximum number of days per year was 34 and the average number was about 14.

It is reasonable to expect the failure rates of Equations 11 or 12 to vary as a function of the fraction of the total number of days in a given month divided by the total number for the years. For example,

$$\lambda_{\text{July}} \sim 114/463 = 0.246$$

Figure 5 indicates the expected variation of both  $\lambda_{\text{fail}}$  and  $\lambda_{\text{nons}}$  on a relative basis. Since Figure 5 represents a 34 year period, we can make a judgment as to whether or not the year being studied is normal by comparing the observed variations in the  $\lambda$ 's with Figure 5. One should note that the shape of Figure 5 will vary with geographical location.

It also must be recognized that lightning is not the sole cause of

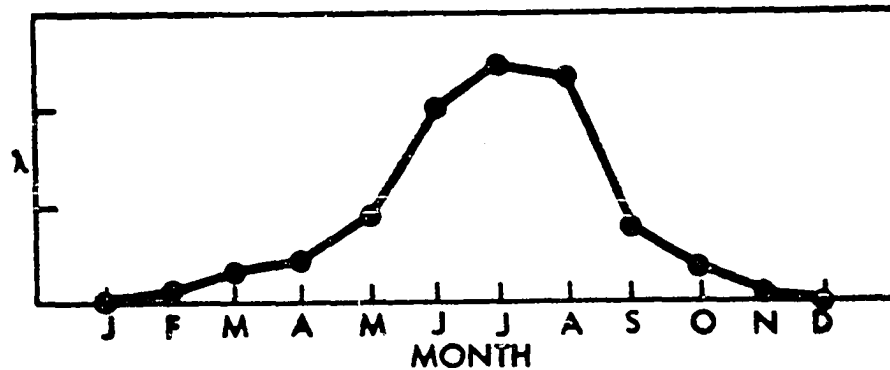


Figure 5. Expected variation of the failure rates by months due to lightning

relay failures. Human errors and communication channel failures are important statistically especially in the winter months when severe weather is relatively rare. Other fault related events include, for example, airplanes hitting the lines and squirrels climbing in the switchgear. However, lightning (or weather in general) is something we have no control over and will always be with us. If one reduces failures due to human errors, for example, this will in turn change the graph of the variation of  $\lambda$  by months analagous to Figure 5. If one reduces all causes of failures except weather to zero, then, in the limit, the graph should approach the shape of Figure 5. Data presented later will indicate the variation of  $\lambda$  for a one year period.

## A NUMERICAL EXAMPLE

The main objective of this numerical example is to provide a clear path from the analytical results to their application to actual operating data. In short, the main results represented by Equations 2, 5, 11 and 12 are applied to the determination of  $q_{fail}$ ,  $q_{nons}$ ,  $Q_{fail}$  and  $Q_{nons}$  for specific power system transmission line relays. Given a sufficient quantity of operating data (many years worth) one can calculate the various  $q$ 's and  $Q$ 's mentioned above. The results based on one year's data are not all expected to have a high confidence level due to the limited number of data points relative to the variance of the data.

Exhaustive operating data were obtained from Commonwealth Edison Company (CECo) for the year 1968. CECo serves approximately the northern one third of Illinois including Chicago and has one of the highest peak loads in the United States. The territory served experiences a wide range of weather conditions typical of the North Central United States.

The operating data used were derived from daily company reports and concerned 128 transmission lines operated at 138kV and higher. Lower voltage equipment was included when it was connected directly to the higher voltage without a circuit breaker. The relays used for protecting lines of 138kV and higher are roughly comparable and are generally of the latest design. Various schemes are used and include permissive, blocking, differential comparison and phase comparison. Most of the lines have a transfer trip (TT) capability using public and private communication channels. TT refers to the requirement of the first relay which responds to the disturbance to trip all of the circuit breakers connected to the

line. -

Since the TT relay is only an auxiliary relay and does not respond to the disturbance directly, it is not included in the reliability statistics. Also excluded are generator, transformer and bus faults. These faults are detected by their own sets of relays and are beyond the scope of this example, but they could be evaluated by the same technique.

Each reported disturbance was noted with regard to the following criteria:

- 1) The nature of the disturbance
  - a) Caused by a short circuit in primary relaying zone
  - b) Caused by a short circuit outside primary relaying zone
- 2) Weather related
- 3) Occurrence of nonselective action
  - a) Fault related
  - b) Security failure (i.e. spurious trip signal)
- 4) Occurrence of failure to operate.

The analyzed data are summarized in Table 4. A great deal of care was taken to exclude redundant information from the raw data. In each of the 306 disturbances a set of transmission line relays responded to what appeared to that device to be a fault. There were a total of 133 actual faults involved in the 306 disturbances. Table 4 shows that a majority of the line disturbances (~60%) are not weather related, and we will assume that these are randomly distributed throughout the year.

Of the 306 disturbances considered, only about 40% were weather related, and all of these except for December and part of January were

Table 4. Summary of relay success and failure for 1968 by months

Month	Line Relay Operations	Weather Related Operations	$\lambda_{fail}$ Failure To Operate	$\lambda_{nons}$ Nonselective Operations	$\lambda_{nons}$ Due to Fault	Other
Jan.	25	10	0	5	5	0
Feb.	4	0	0	2	0	2
March	12	6	0	2	0	2
April	20	9	0	8	5	3
May	25	8	0	8	3	5
June	34	25	0	2	1	1
July	35	8	0	15	1	14
Aug.	67	47	0	13	7	6
Sept.	20	6	0	6	2	4
Oct.	7	1	0	4	2	2
Nov.	16	0	0	9	2	7
Dec.	41	18	0	9	4	5
Totals	306	136	0	83	32	51

caused by lightning. The corresponding weather related failures due to lightning are plotted in Figure 6 (curve 1) along with the data from Table 3 (curve 2).

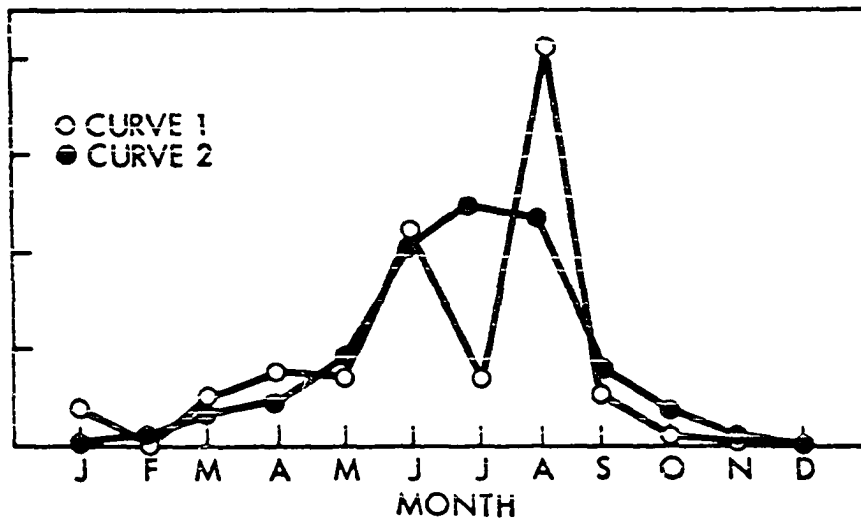


Figure 6. Comparison of 1968 weather related disturbances (curve 1) with the 34 year average incidence of lightning (curve 2)



We can see from Figure 6 the expected correlation between the weather related disturbances due to lightning for 1968 and the incidence of damaging lightning given in Table 3.

#### Determination of $q_{fail}^{\gamma}$ from Operating Data

$$q_{fail}^{\gamma} = q(x_a)q(x_b | x_a)$$

The event  $x_a$  is the occurrence of a fault on line  $\gamma$  and  $x_b$  is a failure to respond to that fault by the set of relays protecting line  $\gamma$ . A useful quantity which is independent of relay placement is  $q(x_b | x_a)$ . Since it represents the probability of failure given a fault,  $q(x_b | x_a)$  is useful for comparisons of different relaying schemes. For line  $\gamma$ ,

$$q(x_b | x_a) = \frac{\text{number of failures of the relays on line } \gamma}{\text{number of faults on line } \gamma}.$$

Note that the denominator is not the total number of operations for line  $\gamma$ , because this would include operations due to security failures. If the relays do not respond to a spurious trip signal, for example, there is no loss of reliability.

The probability of occurrence of a fault  $x_a$  is denoted by  $q(x_a)$ .

For line  $\gamma$

$$q(x_a) = \frac{\text{number of faults on line } \gamma}{\text{number of faults on all lines}}.$$

Therefore, there are two useful quantities related to the "goodness" of the relays with respect to failures to operate on line  $\gamma$  —  $q_{fail}^{\gamma}$  and  $q(x_b | x_a)$ . In order to realize the best return for each dollar spent on protection (with respect to failures to operate) it is desirable to make all the  $q_{fail}^{\gamma}$  for each line equal to each other. This is called the constant hazard approach (10). This approach implies the

use of more reliable relays on more fault-prone lines to obtain a constant  $q_{fail}^Y$ . The other quantity is  $q(x_b | x_a)$  and is useful for comparing different protection systems independent of location. Alternatively,  $q(x_b | x_a)$  is the inherent unreliability of the relay.

If we assume that a computer operated protection system would be more reliable than the relay system it would replace, then the line or lines with the highest faultability,  $q(x_a)$ , represent the best placement of such a system for maximum improvement of reliability with respect to failures to operate.

As shown in Table 4, there were no failures to operate in the presence of a fault on the CECO system for 1968. As a result all  $q_{fail}^Y$  for all lines are identically zero. Also,  $q(x_b | x_a)$  equals zero for those lines on which faults occurred. No data is available for 1968 to determine whether or not  $q(x_b | x_a)$  is zero for the lines which were not faulted. Assume a fictitious failure to operate on line 0000 out of 6 total faults on this line. Then, for the relays protecting line 0000,

$$q(x_a) = \frac{6}{133}$$

$$q(x_b | x_a) = \frac{1}{6}$$

and  $q_{fail}^{0000} = 1/133$  for 1968.

#### Determination of $q_{nons}$ from Operating Data

The event  $y_s^Y$  is a nonselective operation on line  $\gamma$  due to a security failure and  $q(y_s^Y)$  is its probability. The event  $y_a^m$  is an external fault on line  $m$  and  $y_b^Y$  is a nonselective operation on line  $\gamma$  due to the fault

on line  $m$ . If there are  $N$  lines, then for small  $q_{\text{nons}}^\gamma$

$$q_{\text{nons}}^\gamma = q(y_s^\gamma) + \sum_{\substack{m=1 \\ m \neq \gamma}}^N q(y_a^m) q(y_b^\gamma | y_a^m) \quad (13)$$

as in Equation 5.

The quantity  $q_{\text{nons}}^\gamma$  is the probability that the relays protecting line  $\gamma$  will operate nonselectively when either a security failure or a fault external to line  $\gamma$  occurs. The quantity  $q_{\text{nons}}^\gamma$  has two aspects. The first term on the right side of Equation 13 above is not fault related and is due to security failures. For line  $\gamma$ ,

$$q(y_s^\gamma) = \frac{\text{nonselective operations due to security failures on line } \gamma}{\text{total number of nonselective operations on system}} \quad (14)$$

The summation term in Equation 13 contains probabilities concerning the other  $N-1$  lines in the system. For line  $m$ ,  $m \neq \gamma$ ,

$$q(y_a^m) = \frac{\text{number of faults on line } m}{\text{Number of faults on system}} \quad (15)$$

Also, for line  $m$ ,

$$q(y_b^\gamma | y_a^m) = \frac{\text{nonselective operations on line } \gamma \text{ due to faults on line } m}{\text{number of faults on line } m} \quad (16)$$

To compute  $q_{\text{nons}}^\gamma$  in general requires one application of Equation 14 and  $N-1$  applications of Equations 15 and 16. In practice, virtually all nonselective operations will be due to security failures as well as faults on adjacent lines, thereby reducing the number of computations from the general case. Since the quantities of Equations 15 and 16 always occur in product form in the summation of Equation 13, one could take advantage of the cancellation between the numerator of Equation 15

and the denominator of Equation 16. However, the identity of the two quantities will be lost, and  $q(y_b^Y | y_a^m)$  is useful.

Let us compute  $q_{\text{nons}}^Y$  for two lines representing both types of nonselective operations. The necessary system data is listed in Table 5.

Table 5. System data for determining  $q_{\text{nons}}^Y$

Number of faults on the system	Number of nonselective operations on the system
133	83

Line number 2102 operates at 345 KV in central Illinois and is interesting in that all of the nonselective failures were due to security failures and none were due to the 7 adjacent faults. Line 2102 had 6 nonselective operations during the year, all due to spurious microwave transfer trip signals. Using Equation 13 above and Table 5,

$$q_{\text{nons}}^{2102} = \frac{6}{83} + \frac{3}{133} (0) + \frac{2}{133} (0) + \frac{1}{133} (0) + \frac{1}{133} (0) = 0.07 \quad (17)$$

This is an appropriate time to note the large variance associated with the use of Equation 13 and the result of Equation 17 due to the small sample space. Several years data for line 2102 must be incorporated into Equation 13 in order to have a high confidence in computations such as 17.

Line number 1321 operates at 138 KV in central Chicago and is interesting because none of the nonselective operations were the result

of security failures and all occurred simultaneously with faults on adjacent lines.

Table 6. Data for the determination of  $q_{\text{nons}}$

Line m	Number of faults for the year	Nonselective operations of line 1321 due to faults on line m
1323	2	2
1210	1	1

Using Equation 13 and Tables 5 and 6,

$$q_{\text{nons}}^{1321} = 0 + \frac{2}{133} (1) + \frac{1}{133} (1)$$

$$q_{\text{nons}}^{1321} = 0.02$$

A listing of  $q_{\text{nons}}^Y$  for all  $N$  lines will indicate which of the lines have more unreliable relays. For these lines, more insight into the nature of the unreliability and what can be done to improve it can be obtained from a look at  $q(y_s^Y)$  and  $q(y_b^Y | y_a^m)$ . Table 7 below lists the appropriate statistics for line 2102, and Table 8 below lists similar data for line 1321.

From Tables 7 and 8, and Equations 13, 14, and 16 one can conclude:

- 1) Line 2102 is less reliable with respect to nonselective operations because  $q_{\text{nons}}^{2102} > q_{\text{nons}}^{1321}$ .
- 2) All of the nonselective operations on line 2102 are due to security failures because  $q_{\text{nons}}^{2102} = q(y_s^Y)$  for line 2102.

Table 7. Statistics describing  $q_{\text{nons}}$  for line 2102

Line	$q_{\text{nons}}^{2102}$	$q(y_s)$	$q(y_a^Y)$	$q(y_b^Y   y_a^m)$
2102	0.072	6/83		
2101			3/133	0
2105			2/133	0
11608			1/133	0
8014			1/133	0

Table 8. Statistics describing  $q_{\text{nons}}$  for line 1321

Line	$q_{\text{nons}}^{1321}$	$q(y_s)$	$q(y_a^Y)$	$q(y_b^Y   y_a^m)$
1321	0.02	0/83		
1323			2/133	1
1210			1/133	1

- 3) All of the nonselective operations on line 1321 are due to adjacent faults because  $q(y_g^Y) = 0$  for line 1321.
- 4) Line 2102 is not affected by adjacent faults, but the reliability of the line relays could be improved by increasing the security.
- 5) The security of line 1321 is fine, but the line relays operate every time there is a fault on lines 1323 and 1210 since  $q(y_b^Y | y_a^m) = 1$  for all faults on those two lines.

Lines 2102 and 1321 were selected for this example as a vehicle for

explaining the necessary calculations to determine  $q_{fail}^Y$  and  $q_{nons}^Y$  and actually are less reliable than the average transmission line on the CECo system.

#### Determination of $Q_{fail}$ from Operating Data

From the operating data regarding failures to act, CECo appears to have nearly solved this portion of the reliability problem. There were only two failures to act during 1968. These failures were due to a communications channel failure and a wiring error, neither of which are fault related. Consequently,  $\lambda_{fail} = 0$  for the year since the relays never failed to act in the presence of a fault. The unreliability  $Q_{fail}$ , for each set of line relays and for the protection system as a whole for each month and for the year are all equal to zero since there were no failures to act, i.e.  $\lambda_{fail} = 0$  for 1968.

In order to see how a nonzero failure rate will affect the various  $Q_{fail}$  quantities numerically, we will assume a fictitious failure on line 0000 in June. For the system for the year, since  $\lambda_{fail} = 1$  in Equation ii,

$$Q_{fail}^{sys, yr} = 1 - e^{-1}$$

$$Q_{fail}^{sys, yr} = 0.632$$

For the system for each month  $i$ ,

$$Q_{fail}^{sys, i} = 0; i \neq \text{June}$$

and 
$$Q_{fail}^{sys, June} = 1 - e^{-1}$$

$$Q_{fail}^{sys, June} = 0.632$$

Looking next at the  $Q_{fail}$  for each set of line relays on the system,  $Q_{fail}$  for all sets of relays equals zero for the year and each month except those on line 0000. For the relays on line 0000 for the year,

$$Q_{fail}^{0000, yr} = 1 - e^{-1}$$

$$Q_{fail}^{0000, yr} = 0.632$$

For line 0000 for each month  $i$ ,

$$Q_{fail}^{0000, i} = 0 \quad i \neq \text{June}$$

$$Q_{fail}^{0000, \text{June}} = 0.632$$

Since the existence of one failure has such a dramatic effect on the unreliability  $Q_{fail}$ , a word of explanation is in order. In the typical application of classical reliability theory, one is usually interested in the probability of mission success where the length of time involved in the mission is much less than the MTBF (reciprocal of the failure rate). This implies that  $Q$  is near zero, or the mission is no-go. In protective relaying systems, it is common for many failures to occur (both failures to operate and nonselective operations) in a one-year period. As a result, the unreliabilities describing the performance of the relays are often close to unity. The fact that the unreliabilities are close to unity simply shows that the probability of a failure in a one-year or one-month period is very large. The analysis of protection system reliability in terms of classical reliability theory is useful, however, in order to compare existing performance with the performance of a computer operated protection scheme.



### Determination of $Q_{\text{nons}}$ from Operating Data

The more severe reliability problem is obviously the existence of 84 nonselective relay operations. It is obvious, but important to note that there is no protection system provision for back-up to protect against nonselective actions. In fact, the disproportionately large number of nonselective failures is a direct result of overkill in the use of parallel redundancy to lower the probability of failure (see Figure 2, Case B). The conclusion then is to use less not more parallel redundancy to obtain a lower cost of protection, as indicated in Equation 6.

Of direct value to the determination of  $Q_{\text{nons}}$  is a graph showing the variation of the failure rate due to all nonselective actions. The nonselective actions are further divided into two subclasses — fault related and nonfault related (security failures). A statistically significant number of the security failures in 1968 were due to spurious transfer trip signals on common carrier microwave channels (17 for the year, 12 in July alone). Note the large difference in the two curves of Figure 7 for July. Figure 7 shows the total number of failures for each month due to nonselective actions as well as the nonselective failures due only to faults occurring on other lines.

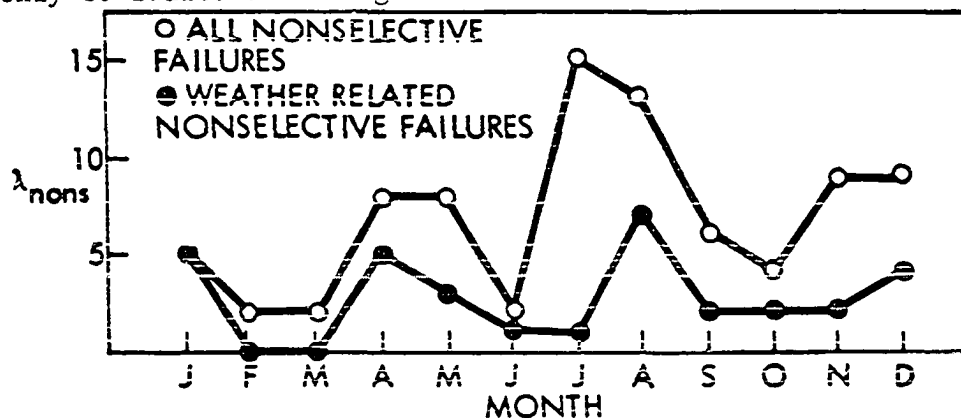


Figure 7. Variation of  $\lambda_{\text{nons}}$  by months for CECO - 1968

The number of nonselective actions for each month and the associated  $Q_{\text{nons}}^{\text{sys, month}}$  from Equation 12 is given in Table 7 below.

Table 9. Unreliability of nonselective action,  $Q_{\text{nons}}^{\text{sys, month}}$  by months. Note that  $Q_{\text{nons}}$  for the best months (0.865) is close to unity

Month	$\lambda_{\text{nons}}$	$Q_{\text{nons}}^{\text{sys}}$
January	5	0.993
February	2	0.865
March	2	0.865
April	8	0.9993
May	8	0.9993
June	2	0.865
July	16	~1.0
August	13	~1.0
September	6	0.9975
October	4	0.98
November	9	0.9998
December	9	0.9998

$Q_{\text{nons}}$  for the year is also given by Equation 12

$$Q_{\text{nons}}^{\text{year}} = 1 - e^{-\lambda_{\text{nons}} T}$$

$$= 1 - e^{-84}$$

$$Q_{\text{nons}}^{\text{year}} = 1 - 10^{-34} \quad (18)$$

One can also determine the  $Q_{\text{nons}}$  for each set of line relays. It is necessary to enumerate the line number and the number of nonselective actions for that line ( $\lambda_{\text{nons}}$ ) then apply Equation 12 again.

For example, let us calculate  $Q_{\text{nons}}$  for line 2102. This line is about 150 miles long, operates at 345 KV and runs north-south through

central Illinois. Table 10 below shows the calculated values for  $Q_{\text{nons}}$  by months for this line.

Table 10.  $Q_{\text{nons}}^{2101}$  for each month for line 2101 for 1968

Month	$\lambda$ 2101 nons	$Q_{\text{nons}}^{2101}$
January	0	0
February	0	0
March	0	0
April	0	0
May	0	0
June	1	0.632
July	1	0.632
August	0	0
September	0	0
October	0	0
November	0	0
December	0	0

For the one-year period of 1968, the "average" unreliability for line 2101 is

$$Q_{\text{nons}}^{2101} = 1 - e^{-2}$$

$$Q_{\text{nons}}^{2101} = 0.865 \quad (19)$$

In other words the probability of at least one nonselective operation on line 2101 during the year is 0.865. Similar calculations could be carried out for each line.

The calculations implied for each set of line relays for each month and for the year will soon become tedious. There must be a lonely

computer somewhere to do these calculations painlessly to 10 nonsignificant figures. Since the data from many years must be crunched, some standard format for data entry and a program to process the data seems prudent.

## COMPUTER OPERATED PROTECTION RELIABILITY

Digital computers and related equipment used in substations to perform protective relaying functions offer many advantages over conventional relaying systems. The flexibility offered by the computer system hardware and software (6) allows one to greatly improve the reliability of the protection system.

As before, the probability of a failure to operate is

$$q_{fail}^Y = q(x_a)q(x_b|x_a)$$

However, for a digital system, the probability of a failure to operate given that a fault has occurred,  $q(x_b|x_a)$ , is independent of  $q(x_a)$ .

Therefore we have

$$q_{fail}^Y = q(x_a)q(x_b) \quad (20)$$

where  $q(x_b)$  represents the probability of failure of the digital system and  $q(x_a)$  is the probability of a fault.

As before, the probability of nonselective operation is

$$q_{nons}^Y = q(y_s^Y) + \sum_{\substack{m=1 \\ m \neq Y}}^N q(y_b^Y|y_a^m)$$

Since the digital system design can anticipate transients and system swings due to adjacent faults (5), the summation term above may be assumed to be zero. Any nonselective operations which occur as a result of faults can then be characterized as security failures and included in  $q(y_s^Y)$ . Therefore we have

$$q_{nons}^Y = q(y_s^Y) \quad (21)$$

where  $q(y_s^Y)$  represents the probability of a nonselective operation of

the digital system due to security failures.

A failure in the digital system can occur either as a hardware failure or as a software failure. The reliability of small process control computers is very good. The MTBF of the central processor and the memory is about 8000 hours (one year = 8760 hours). The probability of a hardware failure is given by classical reliability theory in Equation 8.

$$Q = 1 - e^{-\lambda T}$$

For digital systems, the a priori probability of a hardware failure can be predicted for a time period T based on the time interval between routine maintenance. By expending a sufficient amount of time and money for maintenance the probability of a hardware failure can be made arbitrarily low — an option not available in conventional relaying systems. For the probability of one or more failures for a year period on line  $\gamma$  due to hardware failure,

$$Q_{\text{fail, hdwe}}^{\gamma} = q(x_a) \left[ 1 - e^{-\lambda T} \right] \quad (22)$$

where T is the interval between maintenance checks and  $\lambda = 1/8000$  hours. It is obvious from Equation 22 that  $Q_{\text{fail, hdwe}}^{\gamma}$  can be made quite small.

Similarly, a hardware failure could cause nonselective operations. Other contributors to the nonselective unreliability are communication channels and other security failures. One can write for a one year period

$$Q_{\text{nons, hdwe}}^{\gamma} = q(y_s^{\gamma}) + (1 - e^{-\lambda T}) \quad (23)$$

where  $\gamma$  and  $T$  are defined as in Equation 22 above and  $q(y_s^Y)$  represents the probability of a nonselective operation due to security failures. The amount that  $q(y_s^Y)$  contributes to Equation 23 depends very much on the protection system philosophy and subsequently on the seriousness of a communication channel failure. For example,  $Q_{nons}^Y$ ,  $hdwe$  will be greatly improved if

- 1) Schemes requiring communication channels for information and transfer tripping are not used
- 2) An overreaching-blocking scheme is used based on local information.

A transfer tripping scheme is interconnected by a multiple input OR element (the TT auxiliary relay) which is unreliable. Compounding this is the lack of reliability demonstrated by some of the TT communication channels. An overreaching (responds to faults beyond the end of the line) blocking (blocks a line trip of the fault isn't between the sets of relays) scheme has the dual advantage of providing positive detection of line faults and of graceful degradation should a communications failure occur. Consequently this system will exhibit a lower  $Q_{nons}$ .

These improvements do not come without cost however; the price is an increase in  $Q_{fail}$ ,  $hdwe$ . Similar to Equation 6 regarding the cost of protection per operation, we have Equation 24 below relating the unreliability to protection costs for a time period of one year.

$$\text{Cost}_{\text{year}} = C_{\text{fail}} Q_{\text{fail}} + C_{\text{nons}} Q_{\text{nons}} \quad (24)$$

Based on past relay performance, a trade-off between  $Q_{\text{fail}}$  and  $Q_{\text{nons}}$  would be desirable and economically justifiable.

If the computer software is written properly and debugged (no small task), the unreliability due to a software failure will be zero. Once again, with sufficient expenditure of time and money, the probability of a software related failure can be made arbitrarily small.

Other sources of unreliability now become important. One of the more important ones is the reliability of the power source (3) for the hardware. Again, with sufficient care and dollars, the power source can be made very reliable.



## CONCLUSIONS

The purpose of this thesis is to develop a useful mathematical description of the reliability of existing protective relay systems. In order to solve this problem, two approaches are developed. First, a more conventional approach is developed which gives the probabilities of failure of a set of transmission line relays each time a fault or a nonselective action occurs. This approach meshes very well with intuitive methods of evaluating reliability currently being used. Second, the classical description of reliability as the probability of no failures during a specified time period is developed. This approach is useful to compare existing protection systems with proposed computer operated protection systems. Both descriptions show that the reliability of existing relays has two failure modes — a failure to operate and a nonselective operation.

The conventional approach and the classical approach are then applied to the determination of the several unreliability quantities from actual operating data by the use of a numerical example. This example shows how the main results of the thesis can be used to derive useful indexes of performance. Two of these indexes are  $q_{fail}$  and  $q_{nons}$  which show that when a system disturbance occurs, there is a low probability that the relays will fail to perform correctly. In addition, the calculation of  $Q_{nons}$  shows that the probability is low that a set of relays will perform for a year without at least one failure.

Even though  $q_{fail}$  and  $q_{nons}$  are low, the fact that  $Q_{nons}$  is not sufficiently low shows that further improvement of the reliability is

needed. It is concluded that such improvement could be derived from a computer operated substation protection system. The reliability of proposed digital protection systems is shown to be potentially much better than is possible with conventional protection schemes due to the flexibility offered by the computer software and the inherent reliability of the digital hardware.

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